CHAPTER 7 -- M O M E N T U M

7.1)

a-i.) The *y component* of momentum before the collision is directed upward while the *y component* of momentum after the collision is directed downward. Clearly, momentum in the *y direction* is not conserved through the collision. This is due to the fact that the force at the ceiling is large enough to change the ball's motion over a minuscule amount of time.

a-ii.) Even if friction was acting in the *x direction* during the collision, the frictional force would be small enough and would be applied over a small enough time to allow the momentum-change during the collision to be negligible. As such, momentum is conserved in the *x direction* through the collision.

b.) As the velocity-magnitude is the same just before and just after the collision, energy was not lost and the collision must have been elastic.

c.) The impulse absorbed by the ceiling as a consequence of the ball's collision with it will be *equal and opposite* to the impulse received by the ball from the ceiling. The ball receives no impulse in the *x direction* (its momentum in that direction is the same before as after the collision) but does receive a change of momentum Δp in the *y direction*. Noting that the ball's initial momentum in the *y direction* (i.e., $p_{1,y}$) is upward (i.e., positive) and its final momentum is downward (i.e., negative), we can write:

$$
\begin{aligned}
\Delta p_y &= p_{2,y} - p_{1,y} \\
&= (-mv_2 \cos 30^o) - (+mv_2 \cos 30^o) \\
&= -2(\ m \qquad v_2 \quad \cos 30^o) \\
&= -2(.4\text{ kg})(11\text{ m/s})\ (.86) \\
&= -7.62\text{ nt}\cdot\text{sec.}
\end{aligned}
$$

If the ball's impulse is -7.62 nt·sec, the ceiling's impulse will be +7.62 nt·sec in the *y direction*. This makes sense as the ball's force on the ceiling will be *upward*, hence the *positive sign* on the impulse applied to the ceiling.

d.) The force the ceiling applies to the ball is (-3200 nt)**j**:

$$
\begin{aligned}
\mathbf{F}\ \Delta t &= \Delta \mathbf{p} = -7.62\ \mathbf{j}\text{ nt}\cdot\text{sec} \\
\Rightarrow \quad \Delta t &= \Delta \mathbf{p}/\mathbf{F} \\
&= (-7.62\ \mathbf{j}\text{ nt}\cdot\text{sec}) / (-3200\ \mathbf{j}\text{ nt}) \\
&= 2.4\text{x}10^{-3}\text{ sec.}
\end{aligned}
$$

7.2) Assuming that player #1 is the 60 kg kid and assuming the runners are moving in the *x direction*:

a.) $p_{1,x}$ = (60 kg)(10 m/s) = (600 kg·m/s)
$p_{2,x}$ = (120 kg)(5 m/s) = (600 kg·m/s).

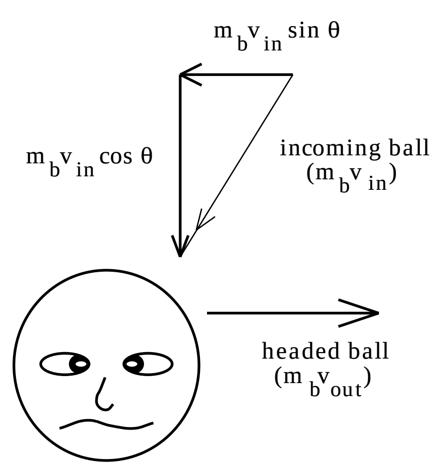
Both players will have the same amount of *momentum*.

b.) $KE_1 = (1/2)m_1{v_1}^2 = .5(60 \text{ kg})(10 \text{ m/s})^2 = 3000$ joules
$KE_2 = (1/2)m_2{v_2}^2 = .5(120 \text{ kg})(5 \text{ m/s})^2 = 1500$ joules.

The players have different amounts of energy.

c.) Energy is what can hurt you. Energy is directly proportional to the mass of the moving object, but it is also directly proportional to the SQUARE of the object's velocity. The lesser amount of energy will be imparted by the larger player moving at the slower speed. It should be noted that although it may be more blessed to give than receive, both parties are going to hurt from the collision (Newton's third law--for every action there is an EQUAL and opposite reaction). Both players will feel the same force. The trick, assuming you want to play a sport predicated on the desire to kill someone, is to make the other guy absorb his blow in a more tender place than where you receive yours. That is, your head impacting his knee is not the way to go.

7.3) The sketch shows the incoming and outgoing ball, complete with momentum magnitudes and momentum components.



a.) It is easiest to do problems like this by first writing out what's happening in the *x direction*, then writing out the momentum equation for what's happening in the *y direction*. Subtracting the balls *incoming momentum* from its *outgoing momentum* in both directions yields the *change of momentum* in both directions. Sooo . . .

<u>x direction</u>:

$$\Delta p_x = p_{out,x} - p_{in,x}$$
$$= (mv_2) - (-mv_1 \sin\theta)$$

$= [(.5 \text{ kg})(18 \text{ m/s}) + (.5 \text{ kg})(25 \text{ m/s})(\sin 30^o)]$
$= 15.25 \text{ kg}\cdot\text{m/s}.$

<u>y direction:</u>

$$\begin{aligned}\Delta p_y &= p_{out,y} - p_{in,y}\\ &= (0) - (-mv_1 \cos\theta)\\ &= (.5 \text{ kg})(25 \text{ m/s})(\cos 30^o)\\ &= 10.83 \text{ kg}\cdot\text{m/s}.\end{aligned}$$

As a vector, $\Delta\mathbf{p} = (15.25\mathbf{i} + 10.83\mathbf{j})$ kg·m/s.

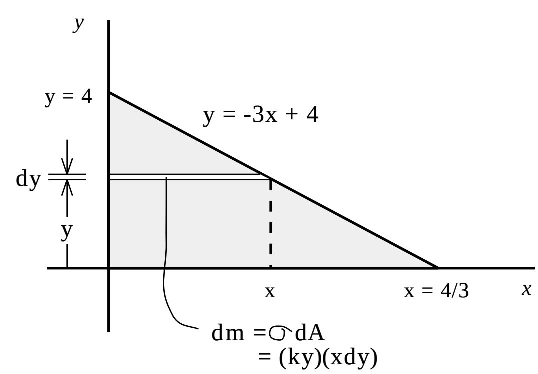
b.) The relationship between *force, change of momentum, and time* is wrapped up in the *impulse equation*. Specifically for the ball:

$$\begin{aligned}\mathbf{F} &= \Delta\mathbf{p}/\Delta t\\ &= (15.25\mathbf{i} + 10.83\mathbf{j}) / (.08 \text{ sec})\\ &= (190.6\mathbf{i} + 135.4\mathbf{j}) \text{ nts.}\end{aligned}$$

This will be equal and opposite the force on your head (N.T.L.).

Note that this is a considerable amount of force. Its magnitude is 270 newtons, or approximately 50 pounds. Also, note that the longer the ball is in contact with the head, the smaller the force is. The moral: from the point of view of your head, it is better to play with an under-inflated ball than an over-inflated ball.

7.4) The sketch shows the structure along with a differential strip of width *dy* a distance *y* units above the *x-axis*. The differential area *dA* of that strip will equal:



$$\begin{aligned}dA &= (\text{length})(\text{height})\\ &= (x)(dy).\end{aligned}$$

Writing x in terms of y can be done using the equation of the line. That is, if $y = -3x+4$, then $x = (-y+4)/3$. As such:

$$\begin{aligned}dA &= (x)dy\\ &=[(-y+4)/3]dy\\ &=[-.33y+1.33]dy.\end{aligned}$$

We need the differential mass associated with the differential area *dA*. We know the area density function is $\sigma = ky$, where k is a constant of magnitude one and units appropriate to the situation (we will assume that $\sigma = y$ from here on as the *k* term will do nothing to the final numerical solution). That means the differential mass *dm* can be written as:

$$
\begin{aligned}
dm &= \sigma dA \\
&= (y)[[(-.33y+1.33)dy] \\
&= (-.33y^2+1.33y)dy.
\end{aligned}
$$

Using the definition of *center of mass*, we can write:

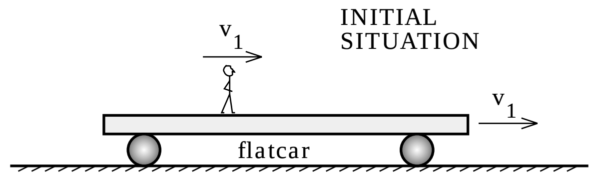
$$y_{cm} = \frac{\int y\,dm}{\int dm}$$

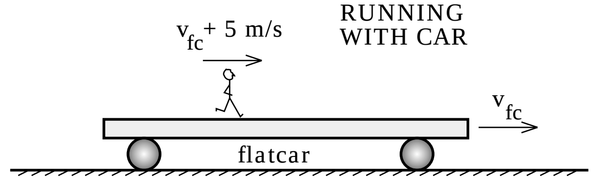
$$
\begin{aligned}
y_{cm} &= \frac{\int_{y=0}^{4} y\left[(-.33y^2 + 1.33y)dy\right]}{\int_{y=0}^{4}\left[(-.33y^2 + 1.33y)dy\right]} \\
&= \frac{\int_{y=0}^{4}\left[-.33y^3 + 1.33y^2\right]dy}{\int_{y=0}^{4}\left[-.33y^2 + 1.33y\right]dy} \\
&= \frac{\left[-.33\left(\frac{y^4}{4}\right) + 1.33\left(\frac{y^3}{3}\right)\right]_{y=0}^{4}}{\left[-.33\left(\frac{y^3}{3}\right) + 1.33\left(\frac{y^2}{2}\right)\right]_{y=0}^{4}} \\
&= \frac{\left[-.33\left(\frac{(4)^4}{4}\right) + 1.33\left(\frac{(4)^3}{3}\right)\right]}{\left[-.33\left(\frac{(4)^3}{3}\right) + 1.33\left(\frac{(4)^2}{2}\right)\right]} \\
&= \frac{7.125}{3.56} \\
&= 2.0 \text{ m.}
\end{aligned}
$$

Note: Students will occasionally try to use the *average value theorem* from Calculus to do a problem like this. That approach works in some cases but not here. Why? Because the *average value theorem* deals only with the outline of the object--it does not take into account the fact that the mass density of the object may not be the same everywhere (i.e., inhomogeneous).

7.5) The bum applies a force to the car; the car applies a force to the bum. As long as the forces are in the direction of the car's motion, all the forces in the direction of motion will be *internal* and momentum will be conserved (note that the velocities are all relative to the stationary track).

a.) Assuming the car is moving in the *x direction*, the bum's mass is m_b, the car's mass is m_c, the initial velocity of both the bum and the car is v_1 in the *+x direction*. After the bum starts running, the final velocity of the car (relative to the ground) is v_{fc} and the final velocity of the bum RELATIVE TO THE GROUND is v_{fc} *+ 5 m/s:*



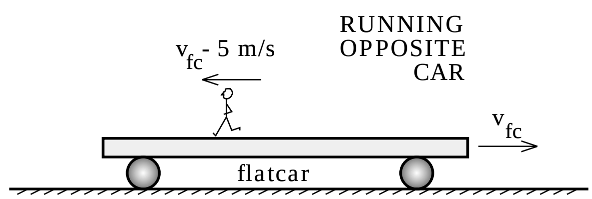


$$\sum p_{init} = \sum p_{final}$$
$$m_b v_1 + m_c v_1 = m_b(v_{fc} + 5 \text{ m/s}) + m_c v_{fc}$$

$$\Rightarrow \quad v_{fc} = [m_b v_1 + m_c v_1 - m_b(5)]/[m_b + m_c]$$
$$= [(60 \text{ kg})(15 \text{ m/s}) + (800 \text{ kg})(15 \text{ m/s}) - (60 \text{ kg})(5 \text{ m/s})]/[60 \text{ kg} + 800 \text{ kg}]$$
$$= 14.65 \text{ m/s}.$$

(It wasn't requested, but this means that $v_b = v_{fc} + 5\ m/s = 19.65\ m/s$).

Does this make sense? Sure it does. The bum pushes off the car making himself go faster. In doing so, he slows the car just a bit.



b.) With the bum running opposite the direction of the car, the bum's final velocity relative to the car is v_{fc} *- 5 m/s*. Following the same steps used in *Part a*:

$$\sum p_{init} = \sum p_{final}$$
$$m_b v_1 + m_c v_1 = m_b(v_{fc} - 5 \text{ m/s}) + m_c v_{fc}$$

$$\Rightarrow \quad v_{fc} = [m_b v_1 + m_c v_1 + m_b(5)]/[m_b + m_c]$$
$$= [(60 \text{ kg})(15 \text{ m/s}) + (800 \text{ kg})(15 \text{ m/s}) + (60 \text{ kg})(5 \text{ m/s})]/[60 \text{ kg} + 800 \text{ kg}]$$
$$= 15.35 \text{ m/s}.$$

(Again, this means that $v_b = v_{fc} - 5\ m/s = 10.35\ m/s$).

Does this make sense? Again, it does. The bum pushes off the car which makes himself go slower relative to the ground. In doing so, he forces the car ahead.

c.) As the bum runs in a direction perpendicular to the car's motion (say, in the *y direction*), nothing changes in the *x direction*--the car's momentum stays the same. Additionally, because there is an *external force* being provided *by the tracks on the train*, momentum is NOT conserved in the *y direction* as the bum picks up speed.

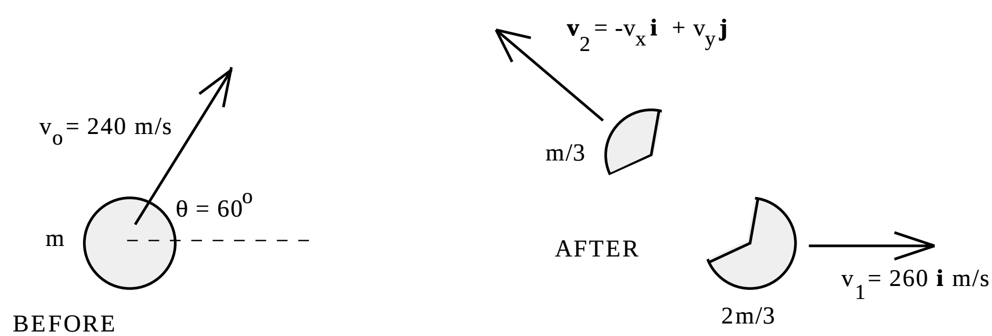
d.) Energy before:

$$\begin{aligned} KE_{bef} &= (1/2)\, m_b v_1^2 + (1/2) m_c v_1^2 \\ &= .5(60\text{ kg})(15\text{ m/s})^2 + .5(800\text{ kg})(15\text{ m/s})^2 \\ &= 96{,}750\text{ joules.} \end{aligned}$$

$$\begin{aligned} KE_{aft} &= (1/2)\, m_b (v_{fc} + 5)^2 + (1/2) m_c v_{fc}^2 \\ &= .5(60\text{ kg})(19.65\text{ m/s})^2 + .5(800\text{ kg})(14.65\text{ m/s})^2 \\ &= 97{,}433\text{ joules.} \end{aligned}$$

Where did the extra energy come from? The bum did work, burning chemical energy in his muscles as he exerted himself. Some of that energy showed itself as *kinetic energy*.

7.6) A sketch of the situation is shown below:



a.) Although there is gravity acting in the *y direction*, the explosion happens so quickly (i.e., Δt is so small) that momentum will be "conserved through the explosion" in all directions. Writing out momentum considerations in both the *x* and *y directions*, and noting that the signs in v_x and v_y are unembedded, we can write:

--In the *x direction*:

$$\sum p_{before,x} \quad = \quad \sum p_{after,x}$$
$$m(240 \cos 60^o) = [(2/3)m]\,(260) + [(1/3)m](-v_x)$$
$$\Rightarrow \quad v_x = 160 \text{ m/s}.$$

Note: This is the *magnitude* of the *x component* of the velocity.

--In the *y direction*:

$$\sum p_{before,y} \quad = \quad \sum p_{after,y}$$
$$m(240 \sin 60^o) \quad = \quad [(1/3)m](v_y)$$
$$\Rightarrow \quad v_y = 623.5 \text{ m/s}.$$

As a vector, final velocity of the second piece is, then,

$$\mathbf{v}_2 = (-160\mathbf{i} + 623.5\mathbf{j}) \text{ m/s}.$$

The magnitude of this vector is 643.7 m/s at an angle of 104.4^o.

b.) With *m* equal to 30 kg, the amount of chemical energy converted to kinetic energy is equal to the increase of kinetic energy (i.e., $\sum KE$). This is:

$$\Delta KE = \quad KE_f \quad - \quad KE_o$$
$$= [(1/2)(2/3)mv_1{}^2 + (1/2)(1/3)mv_2{}^2] - [(1/2)mv_o{}^2]$$
$$= .5[.67(30 \text{ kg})(260 \text{ m/s})^2 + .33(30 \text{ kg})(643.7 \text{ m/s})^2 - (30 \text{ kg})(240 \text{ m/s})^2]$$
$$= 1.87\text{x}10^6 \text{ joules}.$$

This may be an unreasonable figure for a typical explosion, but what do you want from an off-the-wall problem?

7.7) This is a one-dimensional collision problem in which momentum is conserved "through the collision." That means:

$$\sum p_{before} = \sum p_{after}$$
$$m_8 v_8 + m_{10}(0) = (m_8 + m_{10})v_{aft}$$
$$\Rightarrow \quad v_{aft} = m_8 v_8/(m_8 + m_{10})$$

$$= (880/1880)v_8$$
$$\Rightarrow \quad v_{aft} = .468v_8.$$

We need a second expression that has v_{aft} in it. We know something about what happens to the energy in the system after the collision, so using the modified conservation of energy approach for the time interval *after the collision up to the complete standstill point*, we get:

$$\begin{array}{llllll} KE_1 & + \sum U_1 & + \sum W_{ex} & = KE_2 & + \sum U_2 \\ (1/2)(m_8 + m_{10})v_{aft}^2 & + \ (0) & - f_k d & = \ 0 & + \ 0. \end{array}$$

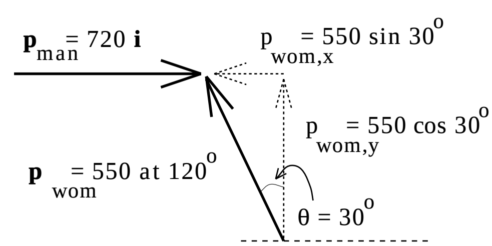
The frictional force f_k is due to m_{10}*'s* brakes locking (m_8*'s* brakes are assumed to remain unlocked). N.S.L. suggests that the normal force on m_{10} in this case is $m_{10}g$ and that the frictional force is $\mu_k N_{10} = \mu_k m_{10}g$. Substituting this into the above expression and solving for v_{aft} yields:

$$(1/2)(m_8 + m_{10})v_{aft}^2 - f_k d = 0$$
$$\Rightarrow \quad (1/2)(m_8 + m_{10})v_{aft}^2 = \mu_k m_{10} gd$$
$$v_{aft} = [2\mu_k m_{10} gd/(m_8 + m_{10})]^{1/2}.$$

Substituting $v_{aft} = .468v_8$ from above into this expression yields:

$$v_{aft} = [2\mu_k m_{10} gd/(m_8 + m_{10})]^{1/2}$$
$$.468v_8 = [2\mu_k m_{10} gd/(m_8 + m_{10})]^{1/2}$$
$$\Rightarrow \quad v_8 = [2(.6)(1000\text{ kg})(9.8\text{ m/s}^2)(1.2\text{ m})/(1880\text{ kg})]^{1/2}/(.468)$$
$$\Rightarrow \quad v_8 = 5.85\text{ m/s}.$$

7.8) The man's initial momentum is in the *x direction*. The woman's initial momentum is in both the *x* and *y direction*. During the collision, the only forces acting are internal to the *two-person system*. This means momentum will be conserved in both the *x* and *y directions* through the collision. Remembering that the two individuals stick together (it is a *perfectly inelastic collision*), we can approach the problem by looking independently at what has happened to the system's momentum in the *x direction*, then *y direction*. Assuming the



final velocity of the two is v_x in the *x direction* and v_y in the *y direction*, we can write:

x direction:

$$\begin{aligned}
\Sigma p_{before,x} &= \Sigma p_{after,x} \\
p_{man\ before,x} + p_{woman\ before,x} &= p_{man\ after,x} + p_{woman\ after,x} \\
m_m(v_{man\ before,x}) + m_w(v_{woman\ before,x}) &= m_m(v_x) + m_w(v_x) \\
(90\ kg)(8\ m/s) + (55\ kg)[(-10\ m/s)(\sin 30^o)] &= (90\ kg)v_x + (55\ kg)v_x \\
\Rightarrow \quad 445 &= 145v_x \\
\Rightarrow \quad v_x &= 3.07\ m/s.
\end{aligned}$$

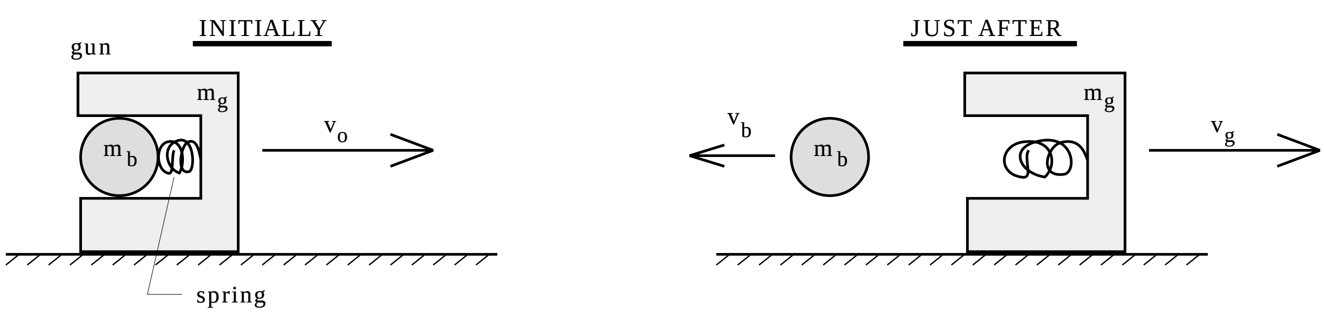
y direction:

$$\begin{aligned}
\Sigma p_{before,y} &= \Sigma p_{after,y} \\
p_{man\ before,y} + p_{woman\ before,y} &= p_{man\ after,y} + p_{woman\ after,y} \\
m_m(v_{man\ before,x}) + m_w(v_{woman\ before,x}) &= m_m(v_y) + m_w(v_y) \\
(90\ kg)(0\ m/s) + (55\ kg)[(10\ m/s)(\cos 30^o)] &= (90\ kg)v_y + (55\ kg)v_y \\
\Rightarrow \quad 476 &= 145v_y \\
\Rightarrow \quad v_y &= 3.28\ m/s.
\end{aligned}$$

The final velocity of the two as a vector will be:

$$\mathbf{v}_{fin} = (3.07\mathbf{i} + 3.28\mathbf{j})\ m/s.$$

7.9) Momentum is conserved "through the one-dimensional firing of the gun" (see sketch on the previous page). As such, we can write:



$$\Sigma p_{before,x} = \Sigma p_{after,x}$$
$$p_{both} = p_{gun} + p_{ball}$$
$$(m_g + m_b)v_o = m_g v_g - m_b v_b$$
$$(2.04\text{ kg})(5\text{ m/s}) = (2\text{ kg})v_g - (.04\text{ kg})v_b$$
$$\Rightarrow \quad v_g = (.04v_b + 10.2)/2$$
$$= .02v_b + 5.1 \qquad \text{(Equation A).}$$

The spring is ideal so no energy is lost in the gun's firing. Using *conservation of energy* through the firing yields:

$$(1/2)m_g v_o^{\,2} + (1/2)m_b v_o^{\,2} + (1/2)kx^2 = (1/2)m_g v_g^{\,2} + (1/2)m_b v_b^{\,2}.$$

Dividing out the *1/2's*:

$$(2\text{ kg})(5\text{ m/s})^2 + (.04\text{ kg})(5\text{ m/s})^2 + (120\text{ nt/m})(.15\text{ m})^2 = (2\text{ kg})v_g^{\,2} + (.04\text{ kg})v_b^{\,2}$$
$$\Rightarrow \quad 53.7 = 2v_g^{\,2} + .04v_b^{\,2}.$$

Substituting *Equation A* in for v_g, we get:

$$53.7 = 2(.02v_b + 5.1)^2 + .04v_b^{\,2}.$$

Expanding yields:

$$.0408v_b^{\,2} + .408v_b - 1.68 = 0.$$

The Quadratic Formula yields:

$$v_b = [-.408 \pm [(-.408)^2 - 4(.0408)(-1.68)]^{1/2}]/[2(.0408)]$$
$$= 3.13\text{ m/s or } -13.13\text{ m/s}.$$

Assuming for the moment that the solution is 3.13 m/s, *Equation A* will give us the gun's velocity:

$$v_g = .02v_b + 5.1$$
$$= .02(3.13) + 5.1$$
$$= 5.16\text{ m/s}.$$

Assuming for the moment that the solution is -13.13 m/s, *Equation A* will give us the gun's velocity:

$$\begin{aligned} v_g &= .02v_b + 5.1 \\ &= .02(-13.13) + 5.1 \\ &= 4.84 \text{ m/s}. \end{aligned}$$

The physical significance of a velocity calculated to be negative, given that we have unembedded the signs on all the velocity terms (hence making them magnitudes), is that the direction of motion has been assumed incorrectly. V_b was assumed to move to the left relative to the ground (i.e., in the negative *x direction*). It is possible we could have been wrong. That is, if the spring had been weak, it would have ejected the ball out the back of the gun, but the ball could have trailed the gun moving slower than the gun but nevertheless to the right in the POSITIVE *x direction*. If that had been the case, we would have computed a negative value for v_b and the negative sign would have told us we had assumed the WRONG DIRECTION for v_b in the first place. That is why we had to at least try the negative velocity value for v_b in *Equation A*.

With v_b negative, the final velocity for the GUN was in the correct direction--to the right--but with LESS VELOCITY MAGNITUDE than it had to start with (it started with 5 m/s velocity--v_g calculates to 4.84 m/s if $v_b = -13.13$ *m/s*). Intuition tells us that this is clearly wrong. Conclusion? $V_b = 3.13$ *m/s* making $v_g = 5.16$ *m/s*.

7.10)

a.) To stop the cart, Tarzan's momentum must be the same as the cart's but opposite in direction. Momentum will be conserved through the collision. If the system is to be brought to absolute rest (i.e., zero momentum) after the collision, we can write:

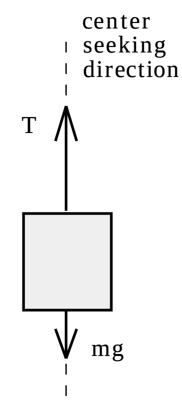
$$\begin{aligned} \sum p_{before,x} &= \sum p_{after,x} \\ (p_j + p_c) + p_T &= 0 \\ (m_J + m_c)v_c - m_T v_{bot} &= 0 \\ (40 \text{ kg} + 190 \text{ kg})(11 \text{ m/s}) - 90v_{bot} &= 0 \\ \Rightarrow \quad v_{bot} &= (230 \text{ kg})(11 \text{ m/s})/(90 \text{ kg}) \\ &= 28.1 \text{ m/s}. \end{aligned}$$

This is the velocity at which Tarzan must move to stop Jane and the cart dead in their tracks.

With this velocity, we can calculate how much energy Tarzan needs at the bottom of his arc. Using *conservation of energy*, we can determine how much of that energy will come from the freefall and how much must come from the run. Using that approach, we write:

$$KE_{top} + \sum U_{top} + \sum W_{ext} = KE_{bot} + \sum U_{bot}$$
$$(1/2)m_T v_{top}^2 + m_T g h_{top} + (0) = (1/2)m_T v_{bot}^2 + (0)$$
$$\Rightarrow \quad v_{top}^2 = v_{bot}^2 - 2gh_{top}$$
$$= (28.1 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(38 \text{ m})$$
$$\Rightarrow \quad v_{top} = 6.7 \text{ m/s.}$$

b.) The f.b.d. to the right shows tension *up* and weight (i.e., *mg*) *down*. Using N.S.L. to sum the forces in the CENTER-SEEKING DIRECTION, we get:



$\underline{\sum F_c}$:

$$T - m_T g = m_T(v^2/r)$$
$$T = m_T g + m_T v_{bot}^2/R$$
$$= (90 \text{ kg})(9.8 \text{ m/s}^2) + (90 \text{ kg})(28.1 \text{ m/s})^2/(19 \text{ m})$$
$$= 4622 \text{ nts.}$$

This is over five times Tarzan's weight of 882 newtons.

7.11) Because this is essentially a collision problem, and because the only force acting (gravitational attraction between the two bodies) is internal to the system, momentum will be conserved in this problem. The difficulty lies in the fact that the planet is huge in comparison to the satellite. That is, the momentum of the planet will only change minusculely due to its size. In short, using *conservation of momentum* really won't work here.

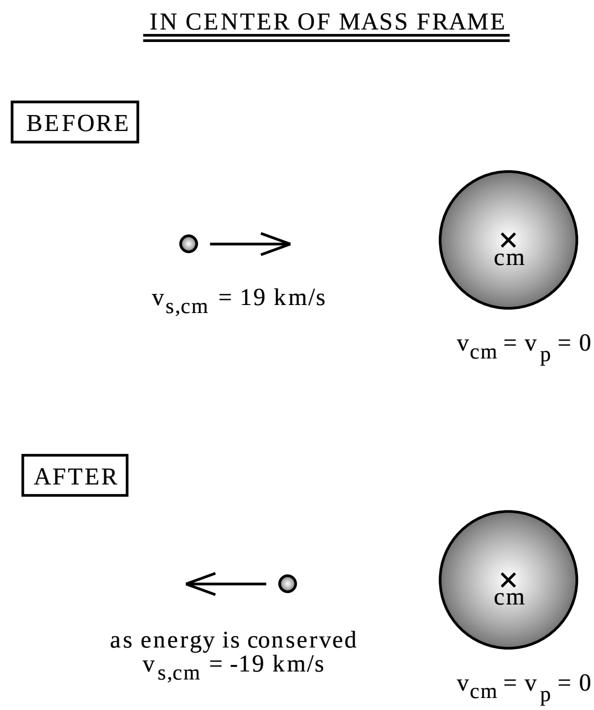
There is a clever way to approach the problem, though. Consider it from a *center of mass frame of reference.*

In the free-space frame (i.e., a frame that is stationary relative to both the planet and satellite), the motion of the center of mass and the motion of the planet will, for all intents and purposes, be exactly the same (almost all of the mass in the system is in the planet). That means that in the *center of mass frame*, the planet will appear stationary. It additionally means that the *satellite's incoming velocity* in that frame (i.e., in the center of mass frame) will be $v_{s,cm} = 7 \text{ km/s} + 12 \text{ km/s} = 19 \text{ km/s}$.

If the so-called collision is elastic, energy will be conserved. That means the satellite will leave the collision with the same amount of energy, hence same velocity, as it entered with . . . FROM THE PERSPECTIVE OF THE *CENTER OF MASS FRAME*.

Relative to space (i.e., the lab frame), the velocity of the center of mass and the velocity of the planet are both 12 km/s. That means the velocity of the satellite, relative to space, will be:

$$
\begin{aligned}
v_s &= v_{cm} + v_{sat.rel.to\ cm} \\
&= 12 \text{ km/s} + 19 \text{ km/s} \\
&= 31 \text{ km/s}.
\end{aligned}
$$



In other words, the satellite will come into the situation moving with velocity 7 km/s and will leave after interacting with the planet with velocity 31 km/s. This slingshot was used by NASA to boost the speed of both Voyager spacecrafts as they passed by Jupiter.